WATERHOUSE

Astronomy
with Engineers' Instruments

Mathematics
A. B.
1907

ASTRONOMY WITH ENGINEERS' INSTRUMENTS

BY

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THESIS FOR DEGREE OF BACHELOR OF ARTS
IN MATHEMATICS

COLLEGE OF SCIENCE
UNIVERSITY OF ILLINOIS

PRESENTED JUNE, 1907

UNIVERSITY OF ILLINOIS

May 31, 1907

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Charles Eugene Waterhouse

ENTITLED Astronomy With Engineers' Instruments

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Arts

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Introduction

It is the purpose of this short treatise on "Astronomy with Engineer's Instruments" to take up, principally, that which can be done by observations with the surveyor's field transit.

The writer does not go into minute detail and demonstrate all that can be done even with the field transit, but has endeavored to take the principal problems and from astronomical observations compute time and azimuth, both roughly and precisely; latitude from the sun as well as from a star by several methods; longitude and the establishment of a meridian.

A large part of the work is from observations of the sun which give the advantage of working in the day light.

Comstock's "Field Astronomy for Engineers" has been the source of the theory obtained and computations contained here in are computed.

by formulas derived in this text. These formulas are based on the astronomical triangle shown on page 31 of the above work. Taking the fundamental formulas of spherical trigonometry.

$$\sin a \sin B = \sin b \sin A.$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos C \cos A.$$

$$\cos a = \cos b \cos c + \sin b \cos C \cos A.$$

and replacing the general symbols by the particular values which are taken from the astronomical triangle

$$a = 90° - h$$

$$b = 90° - \delta$$

$$c = 90° - \phi$$

$$A = t$$

$$B = 180° - A$$

By making the above substitution the astronomical triangle can be applied and these formulas for transforming altitudes and azimuths into declinations and hour angles

$$\cos h \sin A = \cos \delta \sin t.$$

$$\cos h \cos A = -\cos \phi \sin \delta + \sin \phi \cos \delta \cos t.$$

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t.$$

are the results.

In all observing there are some corrections which have to be made. The one which, generally, has to be applied is a correction for

refraction and in observations on the sun a correction for parallax and semi-diameter has to be made. The amount of the correction for refraction is given by the formula

$$Ref. = h'-h = [2.9922]\,B\,\frac{\cot h'}{456+t}$$

in which B = barometer reading and $\underline{t}$ is the thermometer reading at the time and place of the observation. The parallax correction is

$$Par. = 8.8 \cos h'$$

for observations on the sun. The semi-diameter correction may be taken from the Nautical Almanac.

It has been the intention of the writer to make the observations as carefully as possible, eliminating as much instrumental error as could be and in the computations to apply the corrections correctly in order that the most precise results might be obtained.

Art 1. — Determination of Latitude and Longitude from a Map. — To find the latitude and longitude of a station from a map; determine the value of an inch in minutes of arc by measuring the distance between two parallels for latitude and two meridians for longitude and divide this by sixty. Then find the point, for which the latitude and longitude are sought, on the map and measure the distance from it to the nearest parallel and meridian. From this measurement the latitude and longitude can be determined by multiplying the distance in inches by the value, obtained above, for an inch in minutes of arc.

Computation of the latitude of the University of Illinois Observatory by this method.

For latitude, $1^\circ = 4.562$ inches; 1 inch = 13.1'

Nearest parallel is 40° north from the

equator.
Distance from this parallel to the point, located on the map as the observatory, is 0.531 inches

$$0.531 \times 13.1 = 6.'956$$

As the point is north of the 40th parallel this amount is to be added which gives. 40° 6.'956 or 40° 6' 57."4
True latitude is 40° 6' 20."1

For longitude 1° = 3.438 inches or one inch = 17.'1
Nearest Meridian is the 88th west from Greenwich and as the point is west of this Meridian the result will again have to be added. The distance from the Meridian to the point is 0.81 inches

$$0.81 \times 17.1 = 13.'85$$

This added to 88° gives 88° 13.'85 which is 88° 13' 51."1 which reduces to

5 hrs. 52 min 55.4 sec.

West from Greenwich
True Longitude = 5 hrs. 52 min 53.7 sec.

2. Latitude from Sun at Noon.—

The time that the sun approaches a Meridian or its highest altitude can be interpolated from the Solar Ephemeris page 400 of the Nautical Almanac and by measuring the Sun's altitudes just before and after this time or until the altitudes begin to decrease the Latitude can be computed. Taking the greatest reading of the vertical circle as the maximum altitude and substituting in the equation

$$\phi = \delta + z = 90° + \delta - h$$

the Latitude is readily obtained. This equation is derived in Comstock's "Field Astronomy for Engineers" page 32.

The observation is the same as explained in the article on "Time and Azimuth from the Sun" and the motion of the sun in altitude is followed until it begins to diminish and the maximum altitude is

determined by the greatest reading of the vertical circle or by drawing the time-altitude curve.

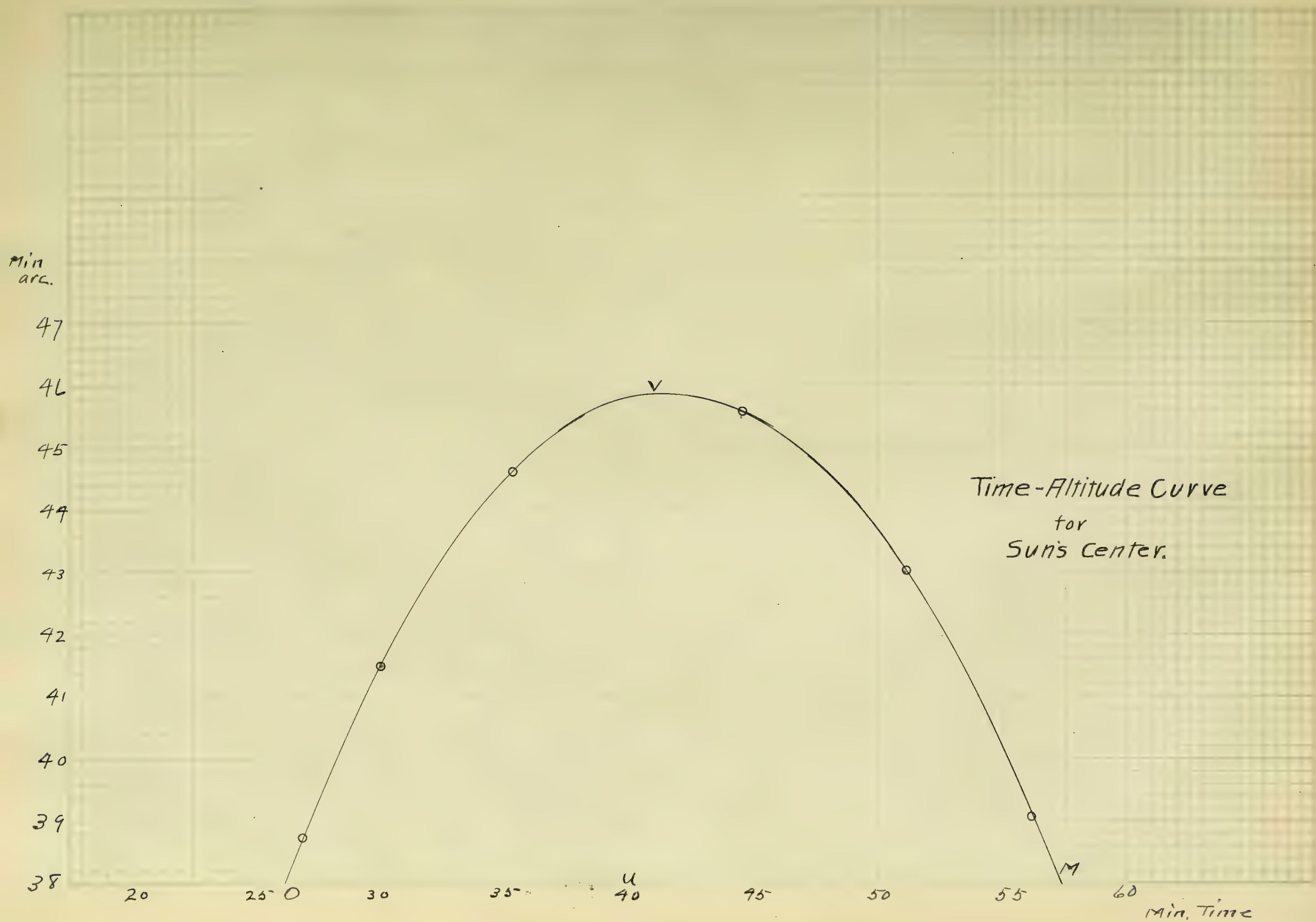
Following is an observation taken at the Observatory of the University of Illinois Oct 21 - 1906.

$\delta = -3^\circ\ 24'\ 41.5''$

$90+\delta = 86\quad 35\quad 18.5$

Transit reading	46°	$45.'8$
Ref-Par.		-0.8
Semi Diam-S-		$-16.$
h	46	29.0
$90+\delta$	86	35.3
ϕ	40	6.3

3. Approximate Latitude from the Sun – In the determination of latitude from the sun by the circummeridian altitudes, there is a difference between each measured altitude and the maximum altitude. This difference is called the reduction to the meridian. These differences can be computed by plotting the readings on cross section paper, using the vertical circle readings in minutes of arc as ordinates and the observed time in minutes of time as abscissae. This curve will be a parabola, whose maximum ordinate will be the vertical circle reading corresponding to the maximum altitude of the sun. Following is the curve of an observation taken by the writer Oct 2-1906



Time-Altitude Curve
for
Sun's Center.
Min
arc.
47
46
45
44
43
42
41
40
39
38
20
25
30
35
40
45
50
55
60
Min. Time
O
U
V
M

This maximum ordinate can be taken directly from the curve or it may be determined more accurately from the area between the parabola and the chord or time axis perpendicular to its axis. This area equals two thirds of the length of the chord multiplied by its distance from the vertex. On the curve on the previous page let $OM = x$ and $UV = y$ then

$$\text{area} = \tfrac{2}{3} x y.$$

As y is what is sought the area and length x should be known. Taking the size of one of the little squares, included in the area, as a unit and counting the number of these little squares in the space the sum will give the total area. The length of x can be measured to the scale of the length of the side of one of these little squares; then the area and the length of x are known. y can now be determined from the equation

$$y = \frac{3A}{2x}.$$

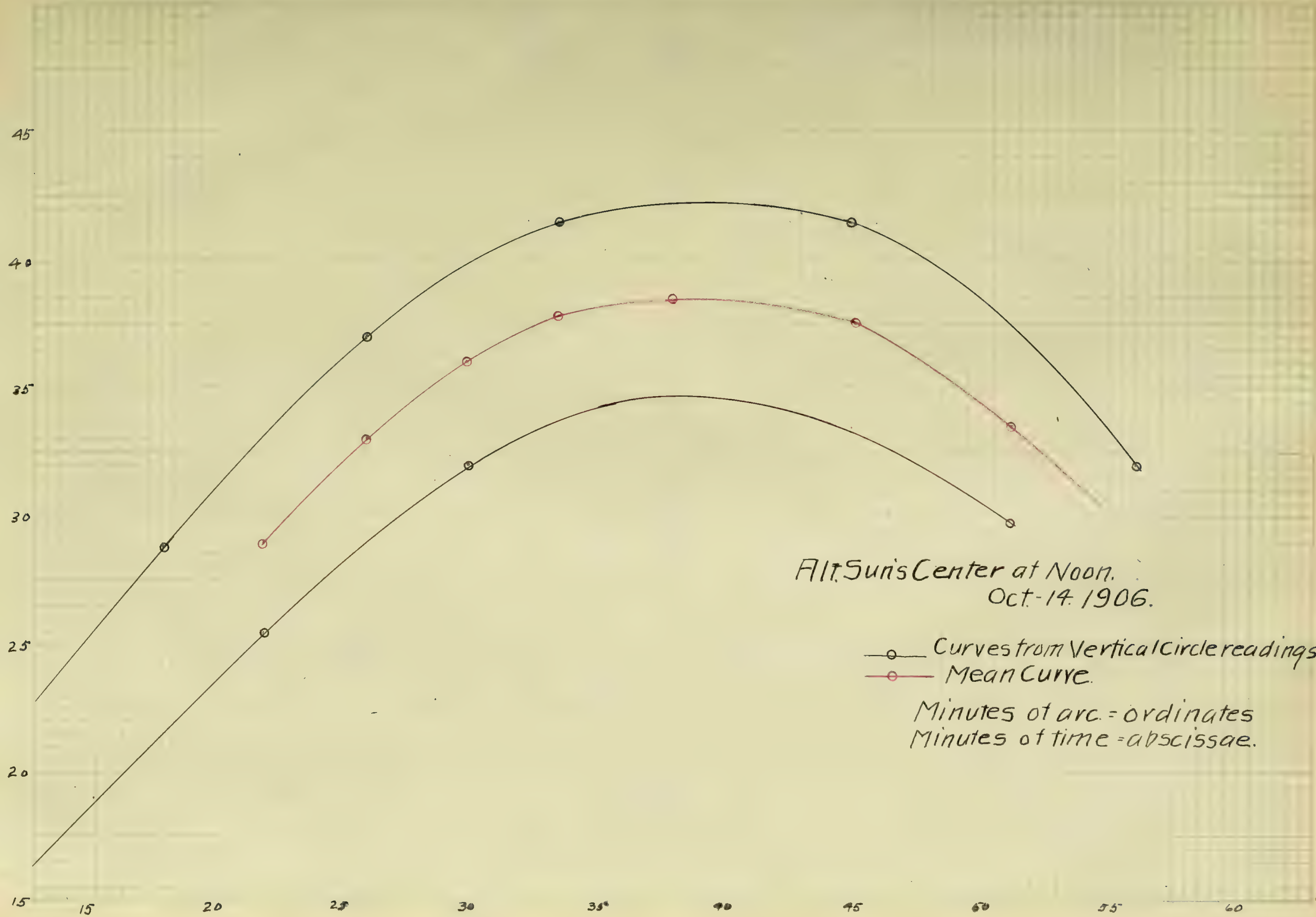
Using this ordinate as the maximum altitude of the sun and the corresponding time, taken from the curve, the latitude

can be computed by the equation

$$\phi = \delta - z = 90^\circ + \delta - h.$$

It can be readily seen that treating the problem thus that each observation contributes its share toward determining the maximum altitude of the sun. The watch correction can be approximately determined from the curve, by noting the point at which the axis of the curve or parabola cuts the time axis and compare with the watch time of apparent noon.

In the computation which follows, in plotting the curve it was found that there was a large index error and instead of but one curve two curves were given with a difference of about eight minutes of arc. In order to get a result a mean curve was drawn and the computations made from it.


Alt. Sun's Center at Noon.
Oct.-14. 1906.
Curves from Vertical Circle readings
Mean Curve.
Minutes of arc = ordinates
Minutes of time = abscissae.
45
40
35
30
25
20
15
15
20
25
30
35
40
45
50
55
60

University of Illinois Observatory

Sunday Oct 14-1906 Alt-Az. instrument.

Watch 3.5 sec. fast. Chas Waterhouse Obs.

Level R	Level L	cir	Watch h	m	s	D °	′	″	vert cir. C ′	″	Mean °	′	″
11.8 9.0	5.7 7.5	R	11	14	19	318	44	54	44	56.5	41	15	4.7
8.9 8.2	7.1 7.9	L	11	18	9	221	28	49.2	28	45.8			
10.6 7.6	5.2 7.8	R	11	21	58	318	34	39.7	34	36.3	41	25	2.2
6.4 7.4	8.6 7.3	L	11	25	45	221	37	3.0	36	58.1			
11.1 7.3	7.8 7.2	R	11	29	50	318	27	59.3	27	54.7	41	32	3.0
6.0 7.0	7.1 6.9	L	11	33	29	221	41	41.0	41	31.9			
11.6 6.9	1.1 6.2	R	11	38	56	318	25	37.6	25	30.9	41	34	25.8
2.2 6.0	7.4 6.3	L	11	49	59	221	41	37.2	41	26.0			
2.2 6.8	8.9 6.6	R	11	51	8	318	30	22.6	30	14.9	41	29	41.2
8.6 6.5	2.1 6.6	L	11	56	23	221	32	17.9	31	54.4			

Computation

$\delta = -7^\circ 59' 45.''6$

$90° + \delta = 82^\circ 00' 14.''4$

Transit reading	41°	38′	39″
Level cor.	+		8.6
cor. Reading = h	41°	38′	47.6
Ref - Par	−		57.0
Semi Diam - S =	+	16	4.3
h	41°	53′	54.9
90 + δ − h = φ =	40°	6′	19.5
True φ	40°	6′	20.″1

Determination of level cor.

N S. $d = 1.''3$

N	S		
R.5.7	11.8		+2.2
		+2.2	
L 8.9	7.1		+2.0
		+1.8	
R 5.2	10.6		+2.8
		+3.8	
L 6.4	8.6		+4.5
		+5.2	
R 2.8	11.1		+5.0
		+4.7	
L 6.0	7.1		+5.2
		+5.8	
R 1.1	11.6		+5.8
L 2.2	7.4		−0.8
		−0.8	
R 8.9	2.2		−0.4
		−0.1	
L 2.1	8.6		−0.1

4. <u>Time from the Altitude of the Sun.</u>— An approximate determination of time can be computed from observations of the sun, when it is from two to four hours away from the meridian. In these observations the altitudes of the sun are measured and the watch time recorded with each observation.

In measuring the altitudes the instrument should be reversed to eliminate all instrumental errors and the sun should be observed with respect to the cross hairs as shown in figures 1 and 2.

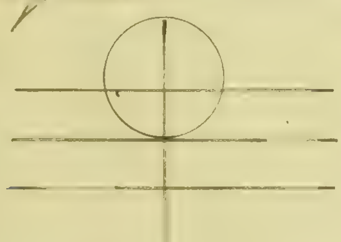


Fig 1

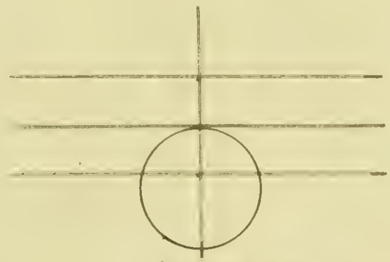


Fig 2

Figure 1 is the observation before reversing the instrument, Figure 2 is after the instrument has been reversed. This is done to eliminate the correction of

semi-diameter. If the observations be taken as shown in figures 3 + 4.

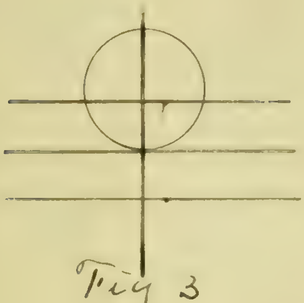


Fig 3

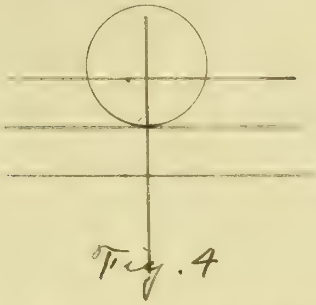


Fig. 4

a correction for semi-diameter must be made in order to bring the center of the sun to the center of the telescope.

The equation for the transformation of coordinates for each observation is

$$\sin h = \sin\phi \sin\delta + \cos\phi \cos\delta \cos t.$$

which gives

$$\cot t = \sec\phi \sec\delta \sin h - \tan\phi \tan\delta$$

from which the hour-angle for each observation can be computed

The time correction in local mean solar time will then be obtained by the equation

$$\Delta T = E + t - T$$

E is the equation of time and can be interpolated from the Solar Ephemeris page 400 of the American Nautical Almanac.

Following is the reduction from observation taken at U. of I. Observatory March 22-1907

University of Illinois Observatory

Friday March 22 – 1907 C E Waterhouse Obs.

cir	Watch h m s	Vert. circle A ° ′		B ° ′	
L	2 55 36	34	9	34	8
R	2 57 42	33	15	33	16

Watch 15.5 sec fast.
Therm 82°
Bar. 29.088 inches

ϕ 40° 6′ 20″

δ 0° 26′ 3.7″

Vert cir-h =	33° 42′ 00″
Ref-Par.	– 1 12.3
Cor h.	33 40 47.7
sin h	9.74384
sec ϕ sec δ sin h	9.86027
tan ϕ tan δ.	7.80517
Subtract	4.44616
Cos t	9.85643
t	44° 4′ 9.2″
E	0h 7m 10.8s
t	2 56 16.6
t + E	3 3 27.4
E of 90th	7 6.1
90th M	6 48.4
T	2 56 39.0
ΔT	– 17.7
True ΔT	– 15.5

sec ϕ	0.11642
sec δ	0.00001
tan ϕ	9.92544
tan δ	7.87973
sec ϕ sec δ	0.11643

5 Time and azimuth from the Sun.—
Time and azimuth are determined from an observation of the sun with a transit. The best time of the day for the observation to be made is when the sun is at least two hours from the Meridian; sometime between eight and ten o'clock in the forenoon or between two and four o'clock in the afternoon.

In order that all instrumental errors may be eliminated, it is best to make two observations by reversing the instrument in altitude and azimuth.

A shade glass may be used over the eye-piece, to break the brightness of the sun or an image of the sun and cross hairs may be thrown on a piece of paper held back of the eye-piece. This gives as accurate an observation as the use of the shade glass.

In taking the observation, the

edge of the sun should be observed and a correction made for semi-diameter. This is done as shown in Figs 5 and 6

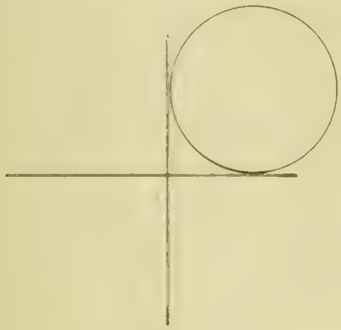


Fig. 5

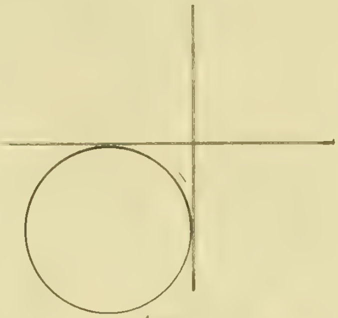


Fig. 6

Fig 5 shows the position of the sun and cross hairs at first observation and Fig 6 shows the position of the same after reversing. If the telescope is provided with stadia hairs the observation may be made using the center of the sun as shown in Fig. 7 and thus avoid the correction for semi-diameter

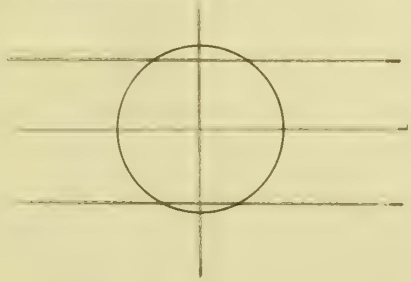


Fig 7

By keeping the instrument as level as possible the level correction is

eliminated

The following is the reduction of an observation taken at the University of Illinois observatory Oct 1 1906. The reduction is by the following equations.

$$K = \sqrt{\frac{\sin S}{\sin(S-a)\sin(S-b)\sin(S-c)}}$$

$$\cot \tfrac{1}{2} A_n = K \sin(S-a)$$

$$\cot \tfrac{1}{2} t = K \sin(S-b)$$

$$\cot \tfrac{1}{2} q = K \sin(S-c)$$

$$\cot \tfrac{1}{2} A_n \cot \tfrac{1}{2} t \cot \tfrac{1}{2} q = K \sin S$$

$$(S-a) + (S-b) + (S-c) = S$$

$$\Delta T = t + E - T$$

$$A = P' - (H' + A_n)$$

T = time of the observation.
H = reading of the horizontal circle of the sun.
n = corrected altitude of the sun's center.
A_n = sun's azimuth reckoned from the north point.
t - the sun's hour angle.
q = is a check angle.
K = an auxiliary quantity.
P' = reading of horizontal circle when sighting at the object.
The refraction and parallax correction

which is used was interpolated from the table on page 62 _Comstock's Field Astronomy_. Mr Comstock derives all the above equations used from the astronomical triangle, on page 65 of the above mentioned text.

University of Illinois observatory Oct 2 - 1906

Station 1, 6 A. Chas Waterhouse obs.

Cir	Object	Watch h m s	Vert Cir A. °	′	B. °	′	Horz. Cir. 14 °	′	15 °	′
L	Beacon						354	12	174	11
L	Sun	2 51 27	27	34	27	35	233	20	53	19
R	Sun	3 4 31	26	11	26	12	53	48	233	47
R	Beacon						174	7	354	8

Watch 27 sec slow.

δ =	$-3°$	28.1′
ϕ	40	6.3
Vert Cir =	26	52.5
Cor		− 1.8
h	26	50.7
a	93	28.1
b	63	9.3
c	49	53.7
2s	206	31.1
s	103	15.6
(s-a)	9	47.5
(s-b)	40	6.3
(s-c)	50	21.9
log sin (s-a)	9.23062	
log sin (s-b)	9.80902	
log sin (s-c)	9.90442	
sum	8.94406	
log sin s	9.98826	

log K^2	1.04420		
log K	0.52210		
log cot ½ Au	9.75172		
log cot ½ t	0.33112		
log cot ½ x	0.42652		
sum	0.51036		
K sin δ	0.51036		
½ Au	60°	29.′7	
Au	120	59.4	
H′	53	33.5	
North	174	32.9	
½t	25	00.6	
(arc) t	50	1.2	
(time) t	3^h	20^m	$\cancel{4}$^{s}
E		−10	31.
(M local) t + E	3	9	33
E 90		7	6
M 90	3	2	27
T	3	2	00
ΔT		+	27
P′	174°	10′	
A	−	22.′9	
True ΔT		+	27^s

6 Azimuth and Latitude from Polaris — If Polaris were at the pole of the heavens it would be a very simple matter to determine the azimuth and latitude for then all that would be necessary to determine azimuth would be to set up the instrument at the station, set the horizontal circle to read zero, sight on Polaris, turn the line of sight toward the direction of which the azimuth is sought and read the horizontal circle and this would be the azimuth of the point sought, from the north point. While sighting at Polaris the reading of the vertical circle would be the latitude of the station for in this case ϕ would equal $\underline{h}$

As Polaris is not exactly at the pole, but is more than a degree away, the latitude and azimuth are not so easily determined, although can be computed from an observation at any time. By making corrections for the amount that the

azimuth and latitude of Polaris differs from the corresponding coordinates of the pole, the determination may be based upon the principles that Polaris is at the pole of the heavens. The values of these corrections are determined from the star's hour angle which comes from the equation

$$t = T + \Delta T + \Delta'.$$

In which t = the hour angle of the star; T = the watch or chronometer time of the observation; ΔT is the correction to be made to reduce T to local mean solar time and Δ' is the difference between the hour angle of the sun and Polaris expressed in minutes and is computed from the equation

$$\Delta' = C(D - E)$$

E is the date that the hour angles of the sun and star are equal; D is the date of the observation and C is the daily increase of the mean sun's right ascension over that of the star, expressed in minutes is

$$C = 4\left(1 - \tfrac{1}{70}\right)$$

$D - E$ is expressed in days.

As the distance of Polaris from the pole varies each year, and the correction for the amount of difference of the azimuth and

latitude of Polaris from the corresponding coordinates of the pole is a function of the latitude of the observer, a correction must be made to the above corrections in order to get the real position of the star with respect to the pole. The coordinates of Polaris can then be written

$$A = 180° + F_1 a$$

$$h = \phi + F_2 b$$

a + b are the corrections for the amount of difference between the azimuth and latitude of Polaris and the corresponding coordinates of the pole. F_1 + F_2 are the corrections necessary to give a + b the required values just for the time and place of the observation. a, b, E, F_1 + F_2 can be interpolated from tables in the back of Comstock's Field Astronomy. F_2 depends upon the year of the observation and F_1 upon the year and the approximate latitude of the station. E F_1 + F_2 need to be interpolated but once a year as they are constant for any place for that length of time.

Following is the reduction of an observation made Oct. 21-1906 at the observatory of the University of Illinois. The Mark used is a button on the agricultural building on the Meridian of Sta. 16

U. of I. Observatory. Sunday. Oct 21-1906

Station 16A   Chas. E. Waterhouse observer.

Cir	Object	Watch h m s	Vert circle A ° ′	Vert circle B ° ′	Horz. circle A ° ′	Horz. circle B ° ′
L	Mark				136 7	316 7
L	Polaris	5 8 8	40 2	40 1	138 4	318 4
R	Polaris	5 14 49	40 4	40 5	318 4	138 4
R	Mark				316 7	136 7

D Oct 21.2   Watch 2.5 sec. slow.

E April 13.6

4(D − E) 762.4

Δ′ 751.2   751.2

+ΔT + 7^m 5^h 15.5^m 5^h 22.1^m

Sum 17 46.7   17 53.1

F_1 0.976   0.976

a +95.′7   +95.′5

$F_1 a$ +92.′9   +92.′2

Az Cir. 138° 4.′0   138° 4.′0

Az Cir − $F_1 a$ 136° 31.′1   136° 31.′8

Mark 136° 7.′0   136° 7.′0

Az of Mark 24.′1   24.′8

Mean 24.5   True az = 24.4

F_2 0.98   0.98

b − 3.′8   −1.′5

$F_2 b$ − 3.′7   −1.′5

Vert. Cir = h 40° 1.′5   40° 4.′5

h − $F_2 b$ = φ 40° 5.′2   40° 6.′0

Mean 40° 5.′6   True 40° 6.′3

I. Accurate azimuth from Polaris – This method of determining azimuth and latitude is more accurate than the one on page 23 because the formulae on which that method is based are only approximate, and an error of one half minute of arc or more may enter during the computation.

In this method all instrumental errors and errors in assumed data can be eliminated. A record of all level readings should be kept. The error due to unequal graduation can be eliminated by turning the horizontal circle from time to time. The observations should be made as rapidly as it is possible for the observer to work without undue haste. Errors of assumed data are eliminated by the method explained later.

Taking the observed time and the right ascension of Polaris its hour angle can be computed, and

from the equations

$$g = \cot\delta \sec\phi$$
$$K = \cot\delta \tan\phi$$
$$-\tan A = \frac{g \sin t}{1 - K\cos t}$$

the true azimuth of the star at the time of observation can be obtained.

t - hour angle.

A - azimuth of star.

The azimuth of the mark is then

$$A' = A + D$$

In which D is the difference of azimuth between the mark and the star at the time T and is assumed to be measured from the star toward the east.

As the precision of A' depends upon D and through A upon the assumed latitude, declination, right ascension and watch correction; that are used in the reduction, whatever error that may come in through any of this data should be eliminated. These errors in the assumed data may be made very small by the selection of Polaris or a star near the pole of the heavens for the observation, as this

makes the factor q very small.

The observation may be made at any time and to get the most precise results two groups of observations should be made and these at different points of its diurnal path; preferably opposite to each other, at an interval of twelve hours. This will almost entirely eliminate all errors in ϕ, δ and α. These errors may be in a great measure eliminated by observations all taken at one time if they are equally divided between stars upon opposite sides and equal distances from the pole.

After the reduction of the hour angles from the formula

$$t = T + \Delta T - \alpha$$

and the constants q and k are obtained the computation of A may be made more easy by the use of Albrecht's Tables which give the logarithm of $\frac{1}{1-x}$ with the argument log x. Letting $\frac{1}{1-x}$ equal F and k cot equal x we have from equations shown above

$$-\tan A = qF \sin t.$$

The method for computing values for F is shown on page 153 of Comstock's Field Astronomy.

The reduction may be made by considering the mean of the number of observations as a single observation taken at the mean of the recorded times. The azimuth thus obtained will not correspond exactly to the observations. To get the highest degree of precision each observation should be reduced independently, thus giving A for each observation and the mean of these values will give very near the correct azimuth.

The following observation was made at the University of Illinois observatory Dec 1-1906 by setting the instrument over a known triangulation point and sighting at an electric light near point near by. This was checked by the method of repetitions.

University of Illinois Observatory Lat. Dec 1 1906

Sta. 16 A — Chas Waterhouse Obs.

Object	Cir.	Watch h	m	sec	Horz circle A °	'	B. °	'	Level W	E
Light	R				282	25	102	24	12.0	10.0
Polaris	R	4	38	15	350	44	170	48	12.0	10.0
Polaris	L	4	44	48	170	46	350	45	10.1	11.9
Object	L				102	25	282	24	16.0	6.0

ϕ = 40° 6' 20" — log sec ϕ = 0.11642 — Watch

δ = 88° 48' 43.6" — log cot δ = 8.31738 — 18 Sec. slow

log tan ϕ = 9.92544

α = 1h 26m 24.6s — log g = 8.43380

log K = 8.24282

	h	m	s	h	m	s
T + ΔT (90th time)	4	38	33	4	45	6
correction		7	6		7	6
T + ΔT (local time)	4	45	39	4	52	12
T + ΔT (sid time)	21	25	9	21	31	42
T + ΔT − α = h	19°	58'	43"	20	5	17
t	299°	40'	45"	301	18	15
log cos t	9.64473			9.71586		
log K cos t	7.93755			7.95868		
sin t	9.93893n			9.93154		
log g	8.43380			8.43380		
F = B,	0.00380			0.00397		
Tan Ao	8.37653			8.37936		
Ao	1°	21'	48"	1	22	20
D	68	24	00	68	21	00
Au	67	2	12	66	58	40
F!	112	57	48	113	1	20

Mean 67 00 26 ; 112 59 34

Check of problem, on preceding page, by repetition

at Station 16 A Button to light by 4 Repetitions

			A			B	
			°	′		°	′
Button	1st reading		00	00		180	00
Light	"	"	243	00		112	59
"	4th	"	41	55		271	54

A 67° 1′ 45″

8. Establishing a Meridian - the Meridian is the true north and south line through a point or station.

A Meridian may be established by calculating the azimuth of some line from the north point with respect to the station through which the Meridian is to run

This azimuth may be found by observation on the sun or a star It may be better to compute the azimuth by the two observations thus obtaining a more accurate result.

When the azimuth of the fixed line is known, set the instrument over the station through which the Meridian line is to pass and sight in the direction of the fixed line and then turn the telescope, toward the north, through a horizontal angle equal to the azimuth computed and this will place

the line of sight on the true north and south line. To fix this line permanently as a reference meridian, fix some solid, stationary point in the line of sight when the telescope is turned on the true north and south line.

We established a true north and south line through the triangulation station 16A, near the University of Illinois observatory, by finding the azimuth of a fixed point and then turning off this azimuth angle toward the north and placed a small white button on the south wall of the Agronomy building of the agricultural department. a distance of about 750 feet due north of station 16A.

9. Meridian Passage of Sun for Time. – The correct time that the sun should cross any Meridian can be determined by interpolating the sun's right ascension from the Solar Ephemeris page 400 of the Nautical Almanac, and making certain corrections; as this time of crossing the meridian changes each day the date of the Month and year will have to be taken into account.

In order to get a correction for time at any place set the instrument up and put the line of sight in the true north and south direction and note the time that the sun crosses the meridian and compare this time with that obtained from the Almanac and the difference is the correction. A plus correction means that the Chronometer is too slow and a

means that it is too fast.
The instrument should be reversed on the sun while crossing the Meridian in order to eliminate all instrumental error. The best way to do this is shown by Figures 8 and 9

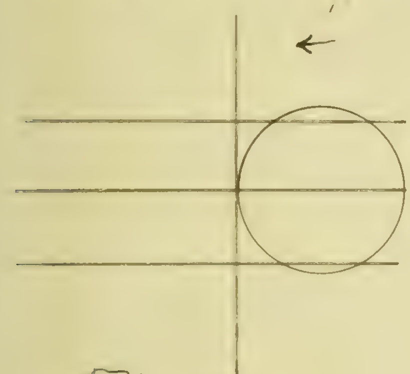


Fig. 8

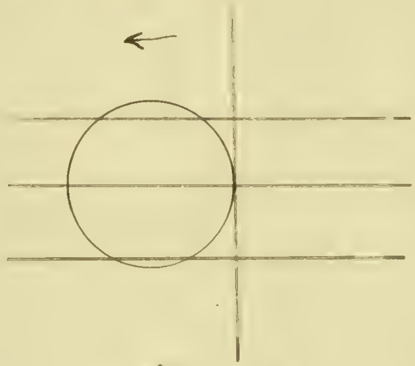


Fig. 9

Fig. 8 shows the sun approaching the Meridian and before the instrument is reversed. Fig. 9 shows the sun leaving the Meridian and instrument reversed. The mean of the two gives the time of the sun's center crossing the Meridian.
In the following computation the instrument was set over Station 16 and was reversed on the sun as shown above.

University of Illinois Observatory
Sunday April 21-1907 Station 16.
Watch 12 Sec. Slow. Chas Waterhouse Obs.

	h	m	s
Time shown by Fig.	11	50	23
Time shown by Fig	11	52	30
Mean	11	51	26.5
Noon	12	00	00
E	-	1	10
E of 10th	-	7	6
	11	51	44
T	11	51	26.5
ΔT		+	17.5
True ΔT		+	12

10. Accurate Latitude from the Sun by Circummeridian Altitudes. — The accuracy of the determination of latitude is greatest when determined by the altitude of the sun observed in the meridian and the computation is very simple; but only one measurement of the altitude can be made by this method and then, not knowing the chronometer correction it is not possible to determine the exact instant when this observation should be taken. If the altitudes are measured near the meridian or at the time when the altitude is near maximum, the observed altitudes can be reduced to the maximum altitude by a simple computation and thus a considerable number of measurements can be made.

The observation should be commenced a few minutes before the sun reaches its maximum altitude and a series of altitudes measured as quickly as possible preferably with an equal number on each side of the

Maximum altitude. These measured altitudes are called circummeridian altitudes.

This reduction is made by the following formulae

$$\phi - \delta_0 + Z - Am + Bn.$$

$$A = \frac{\cos\phi \cos\delta}{\sin Z_0}$$

$$B = A^2 \cot Z_0$$

$$Z_0 = 90^\circ - h_0 \quad \phi - \delta$$

$$m = \frac{2 \sin^2 \frac{1}{2}(t-y)}{\sin 1''}$$

$$n = \frac{2 \sin^4 \frac{1}{2}(t-y)}{\sin 1''}.$$

$$y = \frac{.25465}{A} \Delta\delta = 9.40594 \frac{\Delta\delta}{A}.$$

$$(t-y) = T' - (E - \Delta T + y)$$

δ_0 = the sun's meridian declination.

Z_0 = zenith distance.

δ = the declination corresponding to the hour angle (t)

t = hour angle in seconds of time

$\Delta\delta$ = hourly change in δ given in the Nautical Almanac.

m and n are arbitrary constants obtained from tables in the back of C.L. Doolittle's text called: "A Treatise on Practical Astronomy," computed from the formulae shown above;

y = the hour angle of the sun corresponding to the maximum altitude. It is very small and is in seconds of time

T = the readings of the observed time taken from the watch.

E = the equation of time taken from the Nautical almanac.

It is seen that in order to compute A an approximate value of the latitude is necessary. When measurements of the altitudes are taken on both sides of the maximum altitude a sufficiently close value may be obtained by taking the largest reading or the mean of this reading taken with the ones immediately before and following it, and calling either one of these the maximum altitude of the sun derive an approximate value for ϕ by the formula.

$$\phi = \delta + 90 - h$$

With this value of ϕ, compute A. having A, B is readily obtained. The declination corresponding to the mean of the times of observations is used.

Observation of sun with Alt.-az. instrument

Univ. of I. Observatory Tuesday Dec 4-1906 C.E. Waterhouse obs.

Ci	Level N.	S	Watch h	m	s.	Vert. circle D °	'	"	C '	"	Mean °	'	"
L	10.0 -2.0	8.5 20.7	11	19	28	207	12	32	12	5	27	12	18.5
R	7.5	11.0	11	22	45	332	41	24	42	51	27	17	52.5
L	20.0	-1.5	11	27	51	207	21	23	20	57	27	21	10.0
R	7.8	11.0	11	32	19	332	35	00	34	47.8	27	25	6.1
L	19.0	-1.5	11	37	02	207	26	23	25	59	27	26	11.0
R	7.1	11.0	11	40	33	332	32	13	31	56.	27	27	55.5
L	19.0	-1.5	11	44	21	207	27	14	26	47.2	27	27	0.8
R	8.0	10.0	11	49	01	332	33	15	32	56	27	26	54.5
L	20.0	-2.0	11	51	50	207	25	12	24	52	27	25	2.0
R	8.0	10.0	11	55	03	332	35	51	35	22	27	24	18.5
L	20.2	-2.5	11	57	58	207	21	22	20	58	27	21	13.0
R	7.1	10.2	12	02	01	332	41	39	41	24	27	18	28.5
L	20.0	-2.0	12	04	31	207	15	10	14	51	27	15	0.5
R	7.8	10.2	12	13	3	332	48	29	48	9	27	11	41.0

Level correction.

$d = 1.3''$

N	S				N	S			
10.0 -2.0	8.5 20.7	10.6	10.6	-13.8	8.0	10.0	9.2 10.0	9.6	-12.5
7.5	11.0	9.	9.8	-12.7	20.0	-2.0	10.0	10.0	-13.0
20.0	-1.5	9.2	9.1	-11.8	8.0	10.0	10.4	10.2	-13.3
7.8	11.0	8.7	9.0	-11.7	20.2	-2.5	9.8	10.1	-13.1
19.0	-1.5	8.3	8.5	-11.1	7.1	10.2	9.4	9.6	-12.5
7.1	11.0	8.3	8.3	-10.8	20.0	-2.0	9.8	9.6	-12.5
19.0	-1.5	9.2	8.8	-11.4	7.8	10.2		9.8	-12.7

Level cor.	h			C. h.			T			t - ms		log m.	A m.	n	B n.	h + Am - Bn		
"	°	'	"	°	'	"	h	m	s	ms	s		"		"	°	'	"
-11.8	27	21	10.0	27	20	58.2	11	27	51	15	8	2.6527	354.6	0.48	0.16	27	26	57.6
-11.7	27	25	6.1	27	24	54.4	11	32	19	10	40	2.3490	178.1	0.12	0.04	27	27	53.1
-11.1	27	26	11.0	27	25	59.9	11	31	02	5	51	1.8420	55.6	0.01	0.00	27	26	55.5
-10.5	27	27	55.5	27	27	44.7	11	40	33	2	26	1.0654	4.3	0.00	0.00	27	27	54.0
-11.4	27	27	00.8	27	26	44.4	11	44	21	1	22	0.2244	2.43	[illegible]	0.00	27	26	52.3
-12.5	27	26	54.5	27	26	42.0	11	47	01	6	2	1.8541	51.17	0.01	0.00	27	27	38.2
-13.0	27	25	4.0	27	24	41.0	11	51	30	8	51	2.1864	123.0	0.05	0.02	27	26	53.0
-13.3	27	24	15.5	27	24	2.2	11	55	03	12	4	2.4561	228.6	0.17	0.06	27	27	36.7
-13.1	27	21	13.0	27	20	[illegible]	11	51	[illegible]	11	34	2.6471	[illegible]	0.41	0.1	27	26	[illegible]
-12.5	27	1[illegible]	[illegible]	27	1[illegible]	13.0	12	[illegible]	01	19	2	[illegible]	[illegible]	1.2	0.41	[illegible]	[illegible]	71.
-12.5	27	15	[illegible]	27	17	[illegible]	12	7	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	27	11	41.5	27	11	[illegible]	12	[illegible]	[illegible]	[illegible]	7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

$\varphi = 40°$ [illegible]' [illegible]"
Chron. cor. – 1[illegible]
$\delta = -2[illegible]°\ 11'\ 44.8''$
$\Delta\delta$ – 20.6"
E – 4' 52.4"
Semi diam 16' 15.8"

Z_0 62° 18' 4.6"
Thermometer 46°
Barometer 29.43 inches

Mean h		27°	27'	22"
Semi Diam.			16	15.8
Ref.	–		1	50.8
Par.	+			7.8
h		27	41	54.8
90 + δ		67	48	15.2
φ		40	6	20.4
True φ		40	6	20.1

$$A = \frac{\cos\phi \cos\delta}{\sin z_0}$$

log cos ϕ	9.8836
log cos δ	9.9666
colog sin z_0	0.0529
log A	9.9031
A	0.8

$$B = A^2 \cot z_0$$

log A	9.9031
log A^2	9.8062
log cot z_0	9.7202
log B	9.5264
B	0.336

$$y = \frac{.25465\,\Delta\delta}{A}$$

log .25465	9.4059
log $\Delta\delta$	1.3132 n
colog A	[illegible]
log y	[illegible] n
y	− 6.546

$(t - y) = T' - (E - \Delta T - E \text{ of } 90^{th} + y)$

E	−	9	53.0
ΔT	−		5.0
		9	48.0
E of 90th		7	6.1
		16	54.1
y	−		6.5
		17	0.6
$(E - \Delta T - E \text{ of } 90 + y)$		23 42	59.4

Refraction and Parallax

B = [illegible] t = [illegible]

$h' = 27° \ 27' \ 22''$

$456 + t = 502$

Const.	2.9923		
cot h'	0.2843		
log B.	1.4688		
colog 502	7.2993		
log Ref.	2.0447		
Ref.	110.″8 = 1′ 50.″8		

h'	27°	27′	22.″0
Ref.	−	1	50.8
h''	27	25	31.2
Semi Diam	+	16	15.8
h'''	27	41	47.0

log cos h'''	9.947
log 8.8	0.944
log Par.	0.891
Par.	7.79

11. Latitude from several Stars — Latitude may be computed, from the measurement of the altitude of a star as it crosses the Meridian, by the formula

$$\phi = \delta + z = 90^\circ + \delta - h$$

h is the vertical circle reading.
δ is the declination interpolated from the American Ephemeris.

This gives but one reading for the altitude of the star. A more nearly correct latitude may be had by taking the altitude of several stars as they cross the Meridian and work out the latitude from each one by the above formula and then take the mean as the correct latitude. This gives a much more accurate result. In order to eliminate all instrumental error the instrument should be reversed on each star.

In the following computations

the instrument was set on station 16 and line of sight turned in the true north and south direction and the instrument was reversed on each star.

University of Illinois Observatory

Station 16 — Sunday May 12 - 1907

Chas. E. Waterhouse Obs.

Star.	Cir.	Vert circle A		Vert circle B		δ			Ref.	
α. Ursae Maj	L	112°	9'	112°	10'	+62°	14'	83.9"	0'	23.6"
	R	112	10	112	9					
δ. Centauris	R	35	38	35	39	-14	16	41.2	1	2.2
	L	35	39	35	38					
ξ Hydrae	R	18	34	18	35	-31	20	44.5	2	55.5
	L	18	35	18	34					
β. Leonis	L	65	01	65	00	+15	5	24.6	0	26.8
	R	65	10	65	01					
ε Corvi	R	27	48	27	47	-22	6	20.5	1	53.1
	L	27	44	27	48					

Computation

	α Ursae Maj		δ Centauris		ξ Hydrae		β. Leonis		ε Corvi	
Vertical cir.	112°	4.5'	35°	38.5'	18°	34.5'	65°	00.5'	27°	46.5'
Ref.	-	.4	-	1.4	-	2.4	-	.4	-	1.4
h	112	4.1	35	37.1	18	31.6	65	00.1	27	46.6
90 + δ	152	15.4	75	43.3	58	24.2	105	5.5	67	53.7
φ	40	6.3	40	6.2	40	7.6	40	5.4	40	7.1

Mean φ 40° 6'.3

True φ 40° 6.3

12. <u>The Value of a Level Division</u> — The value of a level division may be determined by having the instrument firmly set up and level, then throw it out of level a certain amount, about one to two degrees. This may be done by sighting on a point when the instrument is level, then throw the instrument [illegible] one or two degrees and bring the point, by this act, back into the center of the telescope by means of the foot screws; this should be taken that the foot screws which carry the telescope to the point are the same [illegible] so that the point is kept in the vertical. If this is not done the telescope can not be directed upon the point chosen when brought back by means of the foot screws. The axis of the instrument will

be tipped through an angle of one or two degrees as a result, as the error for a small angle will be inappreciable. Let this angle be called Y

If the transit is now turned slowly in azimuth the level bubble will run back and forth in its tube and two positions of the instrument will be found where the bubble will stand in the middle of the scale. Any slight turning of the instrument, either way, from these points will cause a slight motion of the bubble.

The best way is to bring the bubble to one end of its tube and the horizontal circle be set to read some multiple of five minutes. Then turn the instrument to each successive 5' readings and record the readings of the extremities of the bubble, until it has traversed the entire length

of the tube, then reverse the movement of the circle and take readings for the same circle settings as before. With reference to the direction of the bubble's motion these two series can be designated "Forward" and "Backward".

The value of the division may then be determined from the formula

$$d = \frac{A' - A''}{2(b' - b'')} \tan Y$$

$A' - A''$ and $b' - b''$ are derived from the readings of the horizontal circle and level respectively.

In the computation which follows the instrument was deflected through an angle of $1° 30'$ by method explained above and the values of the divisions for the two levels were determined. For the upper level the horizontal circle was turned $5'$ for each reading of the extremities of the bubble and for the lower one $15'$ was used.

Value of a Level Division

University of Illinois Observatory April 2-1907

Transit No. 1 Chas Waterhouse Obs.

Circle Reading	Forward E	Forward W	Backward E	Backward W	2b	2(b'-b'')
			Upper Level			
83° 40'	3.0	3.0	3.0	3.0	6.0	
45'	3.5	3.5	3.8	3.8	7.3	
50'	4.0	4.0	4.0	4.0	8.0	
55'	4.2	4.2	4.2	4.2	8.4	
84° 00'	5.0	5.0	4.6	4.6	9.6	3.6
05'	5.2	5.2	5.0	5.6	10.2	2.4
10'	5.5	5.5	5.2	5.2	10.7	2.7
15'	5.9	5.9	5.6	5.6	11.5	3.1
20'	6.2	6.2	6.2	6.2	12.4	
					Mean	3.075
			Lower Level			
1° 10'	2.0	1.4	2.0	[illegible]	[illegible]	
20'	2.5	[illegible]	[illegible]	[illegible]	[illegible]	
30'	3.1	[illegible]	[illegible]	[illegible]	[illegible]	
40'	4.0	[illegible]	4.1	[illegible]	[illegible]	
50'	4.5	4.4	[illegible]	[illegible]	[illegible]	
2° 00'	5.2	4.1	[illegible]	[illegible]	[illegible]	[illegible]
10'	5.7	5.4	[illegible]	[illegible]	[illegible]	[illegible]
20'	6.2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30'	7.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40'	7.8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
					mean	[illegible]

$Y = 1° 30'$

	Upper Level	Lower Level
$A' - A''$	20' = 1200"	50' = 3000"
tan Y	8.4181	8.4181
log $A' - A''$	3.0792	4.2125
colog $2(b'-b'')$	9.5121	3.4771
log d	1.0094	1.1077
d	10.2	12.81
$2d$	20.4	25.6

13 Magnetic Declination — The magnetic declination or the amount the magnetic needle deviates from the north and south line was determined by setting up the transit at Station 16 and sighting on the button or placing the line of sight in the true north and south direction. Both ends of the needle was then read. A series of such readings was taken reversing the instrument between each reading. Then taking the mean as the declination of the needle.

The declination was also derived from the map in the United States Coast and Geodetic Survey Magnetic Declination Tables for 1902.

Magnetic Declination
University of Illinois Observatory. Transit No 2
April 26 1907 3 P.M. C.E. Waterhouse Obs.

South End West of South.		North End East of North	
2°	53′	2°	54′
3°	5′	3°	5′
3°	5′	3°	5′
3°	8′	3°	8′
3°	7′	3°	7′
3°	8′	3°	8′
3°	7′	3°	7′
3°	8′	3°	8′
Mean 3°	5′	3°	5′

From map.

Jan 1 1902	3°	22′
Correction	–	21′
April 1st 1907	3°	1′
Observed	3°	5′

Table of Symbols Used

h	page 6	altitude
Z	6	zenith distance, complement of h.
A	19	azimuth reckoned from the south
A_u	19	azimuth reckoned from the north.
t	15	Hour-angle
δ	6	Declination
α	29	Latitude
S	6	Sun's semi-diameter
T	15	Time shown by Chronometer, whether right or wrong.
ΔT	15	Chronometer correction.
E	15	Equation of time
D	24	Date of an observation.
a	25	Tabular difference of azimuth Polaris and north pole.
b	25	Tabular difference of altitude, Polaris and north pole
E	24	Date of Conjunction, Polaris and mean time.
ϕ	6	Latitude

F_1, F_2	page 25	Factors to trans form a and b into their true local values.
d.	50	value of half a level division
g.K.	28	auxillaries used in computation of azimuth
S	19	auxiliary used in computation of differential refraction.
a, b, c,	19	auxillaries
B.	43	Barometer reading in inches
t	43	Thermometer reading
Ref.	43	Refraction
Par.	43	Parallax
m. n.	34	auxillaries
q	19	auxilliary
Δ'	24	Difference in right ascension Polaris and mean sun.
A, B.	39	Auxillaries
$\Delta\delta$	39	Hourly change in δ.

References.

Comstock - "Field Astronomy for Engineers."

Campbell. - "Elements of Practical Astronomy."

Doolittle. "Practical Astronomy"

American Ephemeris and Nautical Almanac 1906 - 1907.